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The energy spectrum of primary cosmic rays above 10^{12} eV

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Abstract. Recent direct experimental measurements of the primary proton spectrum have given a power law with differential slope $\gamma_d \sim -2{\cdot}75$ for energy less than about 10^{12} eV. Extrapolation of this spectrum gives intensities of the order of those derived from extensive air showers in the region of 10^{18}–10^{19} eV. The data from a variety of experiments which had previously indicated intensities significantly above this spectrum, particularly in the range 10^{14}–10^{16} eV, are re-examined critically. It is concluded that the intensities are indeed higher and that a single exponent is quite inadequate to describe the spectral range 10^{10}–10^{20} eV.

Introduction

The energy spectrum of primary cosmic rays is of interest for two main reasons. Firstly, the shape of the spectrum, and its numerical magnitude have relevance to the astrophysically important problems of where the particles come from, how they are accelerated and how they propagate through space. Secondly, by examining the relationship between the primary spectrum and those of the secondary components in the atmosphere (for example, the muon spectrum) it is possible to draw conclusions about the character of high energy interactions. As an example of the astrophysical significance of the spectrum, Karakula *et al* (1974) have given evidence suggesting that the apparent rapid change of slope of the primary spectrum near 10^{15} eV can be understood in terms of a contribution from pulsars. Concerning the nuclear physical aspect, Wdowczyk and Wolfendale (1973) have presented evidence that characteristics of energetic extensive air showers cannot be understood in terms of an extrapolation of the 'scaling hypothesis' of Feynman (1969) which appears to be successful in explaining at least some of the nuclear physical processes below about 10^{12} eV.

A prominent feature of the primary cosmic ray spectrum accepted for a number of years (see for example the summary by Greisen 1965, figure 1) has been a transition from a differential slope of $-2{\cdot}6$ below $E \sim 3\times10^{15}$ eV to a differential slope of about $-3{\cdot}2$ at higher energies. The bulk of the evidence on which this spectral shape was based came from studies of extensive air showers. Direct measurements of the energy spectrum of primary particles have been made in recent years with balloon- and satellite-borne equipment, ionization calorimeters and spectrometers, mainly at energies below about 10^{12} eV. The exception has been the work with the PROTON series of satellites (eg Akimov *et al* 1970, 1971), which currently extends the measurements to above 10^{15} eV and has a differential exponent of about $-2{\cdot}7$. Unfortunately there has been considerable criticism of this work because it appears to show that the proton component falls very

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rapidly with increasing energy above about 10^{12} eV so that at higher energies heavy nuclei predominate and such a behaviour is inconsistent with conclusions from studies of the muon charge ratio at ground level (eg Ayre *et al* 1972, Ashley *et al* 1973) which show little evidence for a change in mass composition to the limit of measurement, corresponding to primary energies of around 5×10^{13} eV. Furthermore, indirect studies of the mass composition at about 10^{15} eV (see, for example, the summary by Trumper 1970) give no evidence for a change in composition compared with that directly measured at approximately 10^{10} eV (more discussion of these measurements will be given later).

Quite recently a new series of direct measurements has become available using balloon-borne equipment of high quality by the Goddard Space Flight Center group (eg Ryan *et al* 1972). These workers have measured the differential spectrum of primary protons to about 2×10^{12} eV/nucleon and α particles to about 5×10^{11} eV/nucleon. The form given for the proton spectrum is

$$N(E_p) = (2{\cdot}0 \pm 0{\cdot}2) \times 10^4 \, E_p^{\gamma_d} \text{ protons m}^{-2}\,\text{s}^{-1}\,\text{sr}^{-1}\,\text{GeV}^{-1}$$

with $\gamma_d = -2{\cdot}75 \pm 0{\cdot}03$, and that for the α spectrum is

$$N(E_\alpha) = (8{\cdot}6 \pm 1{\cdot}4) \times 10^2 \, E^{\gamma_d} \; \alpha \text{ particles m}^{-2}\,\text{s}^{-1}\,\text{sr}^{-1}\,(\text{GeV/nucleon})^{-1}$$

with $\gamma_d = -2{\cdot}77 \pm 0{\cdot}05$.

The spectra are rather similar to those from the PROTON experiments below 10^{12} eV but they do not show the rapid fall in proton intensity found in the latter work above 10^{12} eV—although it should be noted that the data are not very precise in this region.

What is important, however, is that there is a measure of confirmation for a rather steep spectrum below 10^{12} eV and it is interesting to note that if this spectrum is extrapolated to higher energies with constant exponent ($\gamma_d = -2{\cdot}75$) then the intensities predicted at $E_p \sim 10^{18}$ eV are rather close to those measured in a number of extensive air shower experiments which have moderately high precision in this region. Figure 1

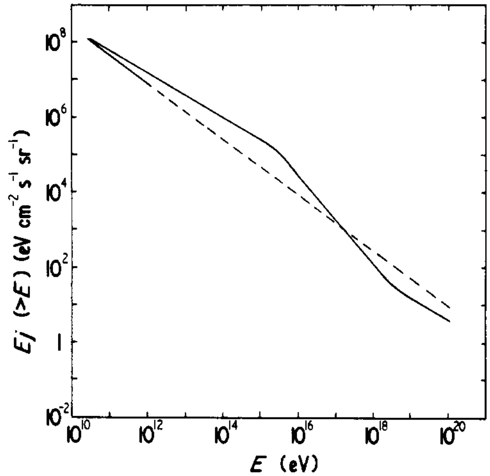

Figure 1. Energy spectrum of primary cosmic ray nucleons over the energy range 10^{10}–10^{20} eV. The full line comes from the summary of Greisen (1965) and is for illustration purposes only—as will be seen later, it has been superseded in parts. The full line and its extrapolated broken line is derived from the proton spectrum of Ryan *et al* (1972), and its extrapolation with constant slope $\gamma_d = -2{\cdot}75$, by multiplying the intensities by a factor 1·3.

indicates the situation. As is indicated in the caption an approximate allowance has been made for heavy nuclei by multiplying the proton intensities by a factor of 1·3; this assumes that the mass composition does not change with energy—a sufficiently good approximation for the present purpose.

In the present work we re-examine the data in the range 10^{12}–10^{18} eV (and beyond), in order to see whether there is still good evidence for the spectral form which takes intensities above the $\gamma_d = -2{\cdot}75$ line and then brings them back to it at about 10^{18} eV.

2. Indirect measurements of the primary spectrum

2.1. General remarks

Three main methods can be identified.

(i) The use of the measured energy spectrum of muons together with a model for high energy interactions and propagation in the atmosphere. This method was used by Brooke *et al* (1964) and later workers. However, in view of the uncertainty in the model at energies above those for which it is known from accelerator experiments (ISR: $1{\cdot}5 \times 10^{12}$ eV) it cannot be used in the present work. In fact, because of the possibility of intra-nuclear cascading in air nuclei (ISR data refer to p–p collisions only) even below 10^{12} eV there are problems. This topic will be examined in a later paper.

(ii) Measurement of the spectrum of nuclear active particles at various depths in the atmosphere. On extrapolation back to the top of the atmosphere the spectrum of primary nucleons can be found.

(iii) Studies of extensive air showers. This is the standard method for measurements above about 10^{14} eV. The experimental techniques are not easy and conversion from measured shower data to primary energy must go by way of a model but for a reasonable range of models the effect on the absolute primary intensity, and more particularly, the slope, is not too large.

In what follows the results from methods (ii) and (iii) will be considered. In the absence of any strong evidence to the contrary it will be assumed that the particles are mainly protons at all energies.

2.2. Nuclear active particles in the atmosphere

A comprehensive survey of all the available data has recently been made by one of the authors (Kempa 1973 and later calculations). The measurements comprised studies with a variety of detectors at different altitudes and with different energy thresholds. Efforts were made to select only data referring to the passage through the detectors of single particles. Assuming that the attenuation length does not vary significantly with energy, over the range $6 \times 10^{11} < E_p < 4 \times 10^{13}$ eV (and is given by the best fit value $\lambda = 115 \pm 5$ g cm^{-2}) the resulting integral nucleon intensities are as shown in figure 2. It is interesting to note that if the attentuation length for nucleons is falling with increasing energy, such as would result from the increase in proton–proton cross section (Amaldi *et al* 1973) taken in conjunction with a constant inelasticity, then the intensities at the higher energies would rise somewhat. The corresponding intensities of nuclei, as distinct from nucleons, are also shown.

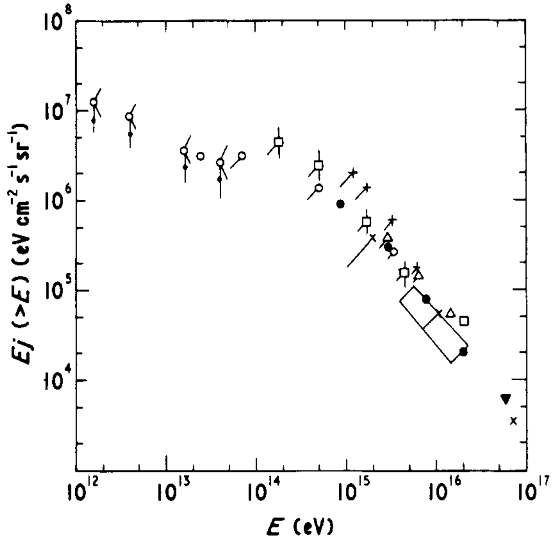

Figure 2. Integral energy spectrum of primary cosmic rays in the range 10^{10}–10^{17} eV. In the analysis it is assumed that the particles are mainly protons; most intensities would rise somewhat if significant fluxes of heavy nuclei were present. ○ Nikolskii (1962); □ Chudakov (referred to by Nikolskii 1962); ● Bradt *et al* (1965); ▼ La Pointe *et al* (1968); + Kameda *et al* (1960); △ Miyake *et al* (1962); × Khristiansen *et al* (1972, electron size data); ◇ Lodz data (see text; upper limit $E^{1/4}$ multiplicity law; lower limit, $E^{1/2}$), Olejniczak (1973); ɸ Kempa (1973): nucleons; ⊄ Kempa intensities multiplied by 1·6 to correspond to nuclei (composition as at 10^{10} eV).

2.3. *Extensive air showers*: $E < 10^{17}$ eV

2.3.1. Electron studies. In many experiments, measurements are made largely on the electron component of the shower and a number of factors indicate that maximum accuracy occurs when the experimental location is near the level of maximum development. For energies in the range 10^{14} eV up to about 10^{17} eV the maximum occurs at aeroplane or mountain altitudes and attention will first be directed towards these measurements.

The highest mountain laboratory in use is that at Mt Chacaltaya in Bolivia and a number of important sets of measurements have been made there. Of particular importance is the work of Bradt *et al* (1965) and this has played an important role in indicating that there may be a rather rapid change of slope in the region of 10^{15} eV (see, for example, the work of M C Bell *et al* 1974). The measurements are also of importance because they give some indication of the height of maximum of the shower and its attenuation beyond the maximum. As will be appreciated, both parameters are of importance in deriving information which relates to the translation from shower size at the detection level to the primary energy. We have adopted as our datum 2 GeV per particle at shower maximum; a number of calculations gives values close to this (mostly in the region of 1·7–1·8 GeV but we argue that the effect of a variety of fluctuation effects not allowed for in the calculations will increase the figure to about 2 GeV/particle).

It is relevant to point out at this stage that there is an absolute lower limit to the energy per electron at shower maximum of 1 GeV per particle for showers initiated by one initial γ ray (eg Rossi 1965). If more than one γ ray is produced and if these γ rays have a distribution of energy (and point of origin), as must be the case in practice, then the lengthening of the cascade immediately causes the number of electrons at shower maximum to fall and the energy per particle to rise.

In all cases the method used is to convert the sizes to what would have been recorded at shower maximum and then use this factor. The points in figure 2 derived from the analysis of the work of Bradt *et al* (1965), which give shower size against depth for different intensity cuts and refer to energies above about 10^{16} eV, are those where the experimental values of size are at depths not far from the shower maximum and we believe that only a little error results from extrapolation of shower sizes to shower maximum.

La Pointe *et al* (1968) have also studied showers at Mt Chacaltaya and this work extended that of Bradt *et al* to higher sizes. The intensities derived by us from the data are shown in part in figure 2 and in part in figure 4.

Before examining data taken at greater depths in the atmosphere, it is necessary to examine the problem of longitudinal development in some detail so that extrapolations can be made for any particular depth to enable an estimate to be made of the shower size which would have been recorded at the level of maximum development. A number of experiments have been carried out at mountain altitudes in which the angular distribution has been studied and these data have been used to estimate the longitudinal development for showers of a fixed primary energy (more specifically, the size is given as a function of slant depth for showers of fixed intensity). Although few enable profiles to be derived all the way from shower maximum to sea level an estimate has been made from the available experiments and the ratio of primary energy to detected size at sea level for vertical showers has been determined in each case. The data are given in figure 3. Comparison with theoretical expectation is also given there.

Proceeding further down into the atmosphere there are measurements on the Pamir mountains by Chudakov *et al* (1960) and Nikolskii *et al* (summarized by Nikolskii 1962). The altitude, 3860 m, corresponds to a depth of 623 $\mathrm{g\,cm^{-2}}$ and corrections have been applied to the data to convert to shower maximum using the conversion factors of table 1. These were calculated using the data which gave rise to figure 3. Insofar as we wish to derive results which are not too dependent on the results from just one experiment conversion factors were also derived from the broken line in figure 3—this was an extrapolation back from 5×10^{16} eV and did not use the data of Bradt *et al* (1965) which were the only source of information in this energy region. The primary intensities shown in figure 2 are represented by diagonal lines—the lower limits arise using the smaller conversion factors from table 1.

Mt Norikura is at a somewhat lower altitude, 2770 m, corresponding to a depth of 730 $\mathrm{g\,cm^{-2}}$ and the measurements from experiments at this location, by Miyake *et al*, (1962, and quoted by Nikolskii 1962) and Kameda *et al* (1960), are also shown in figure 2.

Finally, in this section, mention can be made of the work on electron showers at ground level. The most important feature of this work is the observation of an increase in slope of the size spectrum at $N \simeq 5 \times 10^5$ (see, for example, the work of Khristiansen *et al* 1972, which summarizes the work of the Moscow State University group). The data of figure 3 have been used to make an approximate estimate of the primary spectrum in the energy range 10^{15}–2×10^{17} eV; as with the data from the Pamir experiments lower limits to the intensities have been derived using the lower conversion given in figure 3.

2.3.2. Muon studies. In general, measurements on muons at sea level give rather more accurate primary spectra, a result that follows from the slow attenuation of the muons in the atmosphere and consequent smaller sensitivity to zenith angle (and other factors).

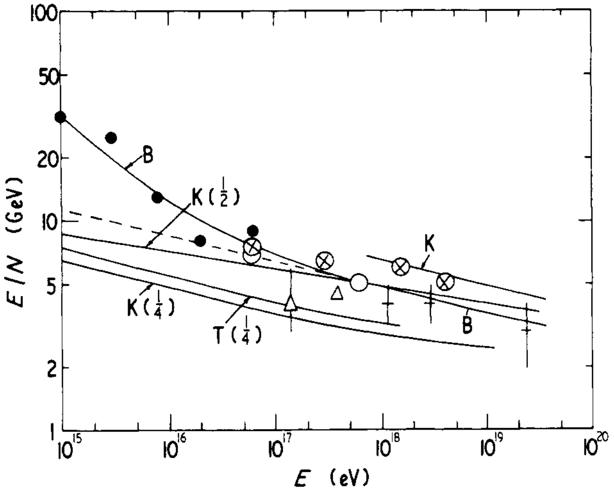

Figure 3. Ratio of primary energy to shower size (E/N) for vertical showers at sea level. It is assumed that the conversion factor at shower maximum is 2 GeV/particle. The curves give the predictions by Kempa (1973) for $E^{1/4}$ and $E^{1/2}$ multiplicity laws (K($\frac{1}{4}$) and K($\frac{1}{2}$)) and T($\frac{1}{4}$) denotes the predictions by Turver (1973) for an $E^{1/4}$ multiplicity law. K denotes the conversion derived from Čerenkov data by Krasilnikov (1973).

The full line (B) is the best estimate drawn through the points; the broken line represents an extrapolation back to lower energies of the straight line fit above 2×10^{17} eV. In this and subsequent figures error bars are given where it was possible to estimate these from the data; the absence of error bars on some points is not to be taken to imply that the uncertainties on the values are necessarily very small. ● Bradt *et al* (1965) (Mt Chacaltaya); △ Clark *et al* (1963) (El Alto); ⊗ Aguirre *et al* (1973) (Mt Chacaltaya); ○ La Pointe *et al* (1968) (Mt Chacaltaya); + Linsley (1973) (Volcano Ranch).

Table 1. Factors to convert from size at measurement level to size at shower maximum N_m for Pamir (Nikolskii and Chudakov) and Mt Norikura (Kameda *et al* and Miyake *et al*) using the longitudinal development curves derived from the data of figure 3 (curve B). The number in parentheses refer to the broken line extrapolation in figure 3.

N_m	10^4	10^5	10^6	10^7
Pamir	3·6	2·6	1·8	1·1
	(2·7)	(2·0)	(1·4)	(1·1)
Mt Norikura		4·0	2·2	1·2
		(3·1)	(1·7)	(1·2)

Many measurements have been made of the density spectrum of muons near sea level and those by the Lodz group. which refer to a threshold muon energy of 5 GeV, are rather precise. Olejniczak (1973) has taken the muon density spectrum together with the experimentally measured muon lateral distribution of Bennett and Greisen (1961) to determine the muon size spectrum. This has then been taken together with the relation between N_μ and E_p given by Giler *et al* (1970, which is close to that of Hillas *et al* 1971 and private communication, de Beer *et al* 1966) to give primary spectra for alternative multiplicity 'laws'; $E^{1/4}$ and $E^{1/2}$. These limiting spectra are also shown in figure 2.

2.4. Extensive air showers: $E > 10^{17}$ eV

2.4.1. Measurements on electrons. This energy range is largely the province of near sea level measurements although some mountain experiments have given data in this region. The latter can be mentioned first. The work of La Pointe *et al* (1968) in this region has already been referred to. Aguirre *et al* (1973, and a later preprint) have recently built a new array at Chacaltaya and the results of their measurements are also given in figure 4. Insofar as the new array has only recently been commissioned, the data should perhaps be taken as tentative (a disquieting feature is that the age parameter, s, of the showers is always measured to be less than unity, eg it is about 0·7 at $N \simeq 3 \times 10^8$ whereas on all the models considered so far it is expected to be rather close to unity). In converting their size at maximum development we have again used 2 GeV per particle.

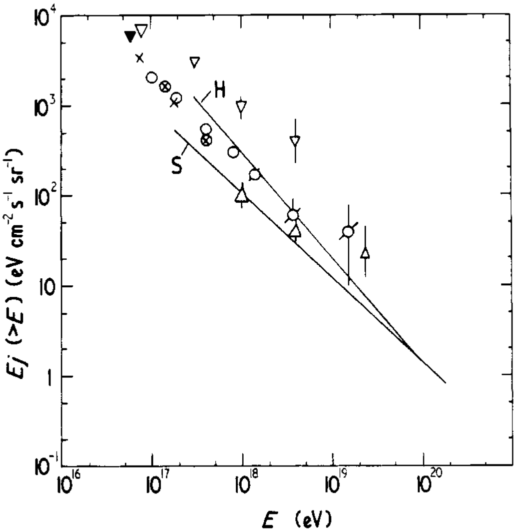

Figure 4. Integral energy spectrum of primary cosmic rays in the energy range 10^{17}–10^{20} eV. ▽ Aguirre *et al* (1973); ○ Krasilnikov (1973), recalculated by us; ⊗ Clark *et al* (1963); ▼ La Pointe *et al* (1968); △ Linsley (1973, Volcano Ranch data of 1963 preliminary re-analysis by Linsley); × Khristiansen *et al* (1972); S Sydney experiment (C J Bell *et al* 1974); H Haverah Park experiment (Edge *et al* 1973).

Another comparatively new array is that, near sea level, at Yakutsk (Krasilnikov 1973). The energy spectrum quoted from this array was derived using the E/N conversion shown in figure 3 labelled K. However, we feel that, following the other data shown in figure 3, the conversion is too high and we have recalculated their spectrum using our best fit N/E curve of figure 3. The new data are given in figure 4 where actual points, with their errors (statistical) are shown.

The classical work at sea level was that of Linsley in the 1960's using the array at Volcano Ranch. These data are currently being re-analysed (Linsley 1973, and a later preprint) and the complete re-analysis is not yet available but preliminary results give the intensities shown in figure 4.

2.4.2. Measurements on muons. At the energies considered here there seems little doubt that measurements involving muons are superior to those concerning electrons. Two important experiments are in progress at the present time and these have been well documented: the Sydney muon experiment (C J Bell *et al* 1974, and earlier references given therein) and the Haverah Park array (which derives its response from a mixture of muons and electrons: the latest work is that of Edge *et al* 1973). The most recent data from these two experiments are given as lines in figure 4. Of particular note is the great care that has been taken with processing the data and the precision with which the slope of the spectrum is claimed to be known:

$$\gamma_d = \begin{cases} -3{\cdot}17 \pm 0{\cdot}03 & \text{for } 3\times10^{17} < E_p < 10^{19}\text{ eV} \\ & \text{Haverah Park (Edge \textit{et al} 1973)} \\ -2{\cdot}96 \pm 0{\cdot}02 & \text{for } 1{\cdot}8\times10^{17} < E_p < 1{\cdot}8\times10^{20}\text{ eV} \\ & \text{Sydney (C J Bell \textit{et al} 1973).} \end{cases}$$

In fact, as will be seen, the two slopes are not statistically consistent but this arises from the use of different interaction models in the process of relating muon densities to primary energies. The model used by Edge *et al* is a conservative one (Hillas *et al* 1971, involving an $E_p^{1/4}$ multiplicity law and very similar to that used by the present authors in a number of publications—see, for example, the work of de Beer *et al* 1966). The model adopted in the latest Sydney work follows the calculations of Goorevich and Peak (1973) and an essential feature is that the multiplicity law approaches $E_p^{0\cdot3}$ at high energies and, more important, lateral distributions are taken for the muons which are derived from other experimental data at lower energies and which appear to us not to be in good agreement with conventional model predictions.

It is interesting to note that use of the same model in the analysis of the Sydney and Haverah Park experiments gives primary spectra which are virtually identical (C J Bell *et al* 1974).

3. Conclusions

It was stated in the introduction that the object of the present work was to see whether all the experimental data could be explained in terms of a simple spectrum of exponent $\gamma_d = -2{\cdot}75$ for $10^{11} < E_p < 10^{20}$ eV. It is concluded that this is highly unlikely, for the following reasons.

(i) There is a wealth of independent EAS data in the range 10^{14}–10^{16} eV indicating that the primary intensities are a factor of 10 above expectation on the basis of $\gamma_d = -2{\cdot}75$.

(ii) Both the mountain data and those at sea level suggest a change of slope by an amount $\Delta\gamma \simeq 0{\cdot}6$ in the energy range 3×10^{15}–10^{16} eV (eg Bradt *et al* 1965, Khristiansen *et al* 1972).

(iii) The most precise spectral data of all at energies above 10^{17} eV give values of γ_d inconsistent with $-2{\cdot}75$ (eg the Haverah Park work which appears to give the best description in this energy region gives $\gamma_d = -3{\cdot}17 \pm 0{\cdot}03$ for $3\times10^{17} < E_p < 10^{19}$ eV).

Finally, if the ‘peak’ is in fact real, it would appear to give rise to a nucleon spectrum at ground level having some measure of flattening above an energy of about 10^{12} eV. It is conceivable that the characteristics of this form observed recently by Baruch *et al* (1973) have some connexion with this hypothesis rather than with the presence of a ‘new’ particle.

Acknowledgments

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